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Analysis and Application of Queuing Theory in Supermarkets

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ABSTRACT: Waiting lines or queues are a common phenomenon in life, especially in the province of organizations that are for profit making. Queues are common in such places as petrol or filling stations, supermarkets stores, clinics, hospitals, motor parks, manufacturing firms, to mention a but a few. An interesting aspect of queuing process resides in the measures of its system's performance, especially in terms of average service rate, systems, utilization and the costs implied for a given capacity level. This paper examines efficient queue management in XYZ supermarket as case study. One of the expected gains from studying queuing systems is to review the efficiency of the models in terms of utilization and waiting length, hence increasing the number of queues so customers will not have to wait longer when servers are too busy. In other words, trying to estimate the waiting time and length of queue(s), is the aim of this study. Through this, we will try to estimate that at given time how many servers will be required so that the total cost is optimized.

I. INTRODUCTION

Delays and queuing problems are most common features not only in our daily-life situations such as at a bank or postal office, at a ticketing office, in public transportation or in a traffic jam but also in more technical environments, such as in manufacturing, computer networking and telecommunications. They play an essential role for business process re-engineering purposes in administrative tasks. "Queuing models provide the analyst with a powerful tool for designing and evaluating the performance of queuing systems." Whenever customers arrive at a service facility, some of them have to wait before they receive the desired service. It means that the customer has to wait for his/her turn, may be in a line. Customers arrive at a service facility with several queues, each with one server (sales checkout counter). The customers choose a queue of a server according to some mechanism (e.g., shortest queue or shortest workload). This paper is the review of queuing theory and for empirical study the sales checkout service unit of a supermarket is chosen as an example. The main purpose of this paper is to review the application of queuing theory and to evaluate the parameters involved in the service unit for the sales checkout operation in the Supermarket. Therefore, a mathematical model is developed to analyze the performance of the checking out service unit. Two important results need to be known from the data collected in the supermarket by the mathematical model: one is the 'service rate' provided to the customers during the checking out process, and the other is the gaps between the arrival times (inter arrival time) of each customer per hour. In order to get an overall perspective of the customer's quality of service, the questionnaires which indicate the result in percentages, are also used to get the evaluation from the customers directly.

II. RELATED WORK

Anthony Egwe, J.U.J. Onwumere, Obiamaka P. Egbo[1] in their paper 'Efficient Queue Management in Supermarkets: A Case Study of Makurdi Town, Nigeria' have examined the Efficient Queue Management in Nigeria with Makurdi as case study. The analysis reveals that on the aggregate, queue management in Makurdi town, the capital of Benue State is grossly inadequate, inefficient and ineffective for the most part. The paper has consequently made suggestions to

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help mitigate the prevailing queuing problems in Makurdi town, which will be relevant to many cities in developing countries having similar challenges.

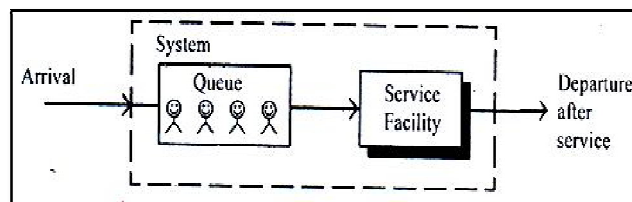
Priyangika J.S.K.C and Cooray T.M.J.A[2] in their paper ‘Analysis of the Sales Checkout Operation in Supermarket Using Queuing Theory’ explain the analysis of Queuing systems for the empirical data of supermarket checkout service unit using queuing theory. The main purpose of this paper is to review the application of queuing theory and to evaluate the parameters involved in the service unit for the sales checkout operation in the Supermarket. Therefore, a mathematical model is developed to analyze the performance of the checking out service unit.

S. Vijay Prasad , V.H. Badshah , Tariq Ahmad Koka[3] in their paper ‘Mathematical analysis of single queue multi server and multi queue multi server queuing models: comparison study’ derived the total cost with assumption of certain Waiting cost in both cases, and proved that the expected total cost is less for single queue multi server model as comparing with multi queue multi server model. Mathematically it has been proved by the examples.

Dr. Engr. Chuka Emmanuel Chinwuko, Ezeliora Chukwuemeka Daniel , Okoye Patrick Ugochukwu, Obiafudo Obiora J.[4] in their paper ‘Analysis of a queuing system in an organization (a case study of First Bank PLC, Nigeria)’ analyzed and concluded that the number of their servers was not adequate for the customer’s service. It was observed that they need 5 servers instead of the 3 at present. It suggests a need to increase the number of servers in order to serve the customer better.

III. PROPOSED METHODOLOGY AND DISCUSSION

M/M/1 queuing model:



M/M/1 queuing model means that the arrival and service time are exponentially distributed (Poisson process). For the analysis of the cash transaction counter M/M/1 queuing model, the following variables will be investigated:

λ : The mean customers arrival rate

μ : The mean service rate

$\rho = \frac{\lambda}{\mu}$: utilization factor

Probability of zero customers in the bank:

$$P_0 = 1 - \rho$$

The probability of having n customers in the bank:

$$P_n = P_0 \rho^n$$

The average number of customers in the bank:

$$L_s = \frac{\rho}{1-\rho} = \frac{\lambda}{\mu-\lambda}$$

The average number of customers in the queue:

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Vol. 6, Issue 9, September 2017

$$L_q = L \times \rho = \frac{\rho^2}{1-\rho} = \frac{\rho\lambda}{\mu-\lambda}$$

The average waiting time in the queue:

$$W_q = \frac{L_q}{\lambda} = \frac{\rho}{\mu-\lambda}$$

The average time spent in the bank, including the waiting time

$$W_s = \frac{L}{\lambda} = \frac{1}{\mu-\lambda}$$

M/M/s queuing model:

λ : The mean customers arrival rate

μ : The mean service rate

$\rho = \frac{\lambda}{s\mu}$: utilization factor

Probability of zero customers in the bank:

$$P_0 = \left[\sum_{n=0}^{s-1} \frac{(s\rho)^n}{n!} + \frac{(s\rho)^s}{s!(1-\rho)} \right]^{-1}$$

The probability of having n customers in the bank:

$$P_n = P_0 \rho^n$$

The average number of customers in the bank:

$$L_s = L_q + \frac{\lambda}{\mu}$$

The average number of customers in the queue:

$$L_q = P_s \frac{\rho}{(1-\rho)^2}$$

The average waiting time in the queue:

$$W_q = \frac{L_q}{\lambda} = P_s \frac{1}{s\mu(1-\rho)^2}$$

The average time spent in the bank, including the waiting time:

$$W_s = \frac{L_s}{\lambda} = W_q + \frac{1}{\mu}$$

Assumptions:

Data for this study were collected from XYZ supermarket. The methods employed during data collection were direct observation and personal interview and questionnaire administering by the researcher. Data were collected for 1 week. The following assumptions were made for queuing system which is in accordance with the queue theory. They are:

1. Arrivals follow a Poisson probability distribution at an average rate of λ customers per unit of time.
2. The queue discipline is First-Come, First-Served (FCFS) basis by any of the servers. There is no priority classification for any arrival.
3. Service times are distributed exponentially, with an average of μ customers per unit of time.
4. There is no limit to the number of the queue (infinite).

International Journal of Innovative Research in Science, Engineering and Technology

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Vol. 6, Issue 9, September 2017

5. The service providers are working at their full capacity.
6. The average arrival rate is greater than average service rate.
7. Servers here represent employees of the supermarket.
8. Service rate is independent of line length; service providers do not go faster because the line is longer.

Comparison of single queue-multi server verses multi queue-multi server:

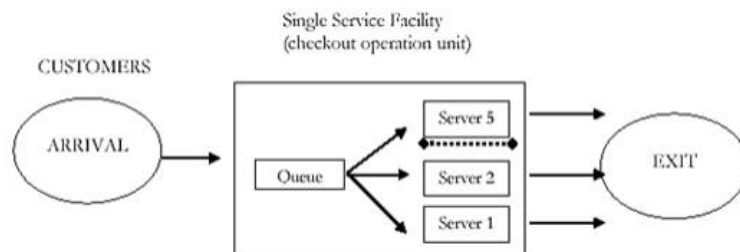


Figure 1. Single Stage Queuing Model with Single-queue and Multiple Parallel Servers

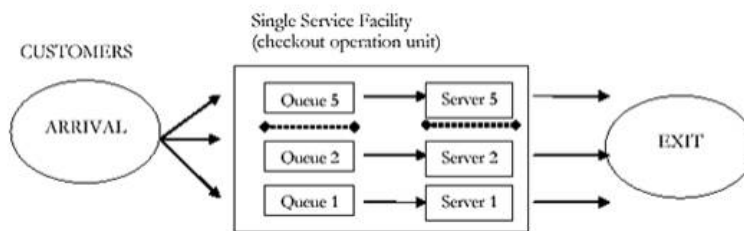


Figure 2. Single Stage Queuing Model with Multiple Queues and Multiple Parallel Servers

Figure 1 describes single stage queuing model with single queue and multiple parallel servers. For this, we will use $M/M/S$ queuing system.

Figure 2 describes single stage queuing model with multiple queues and multiple parallel servers. For this, we will use $M/M/1$ queuing system.

If we consider S number of checkout stands in the supermarket, and the customer's arrival rate is λ and the service rate of each checkout stand is μ , if we try to find the values of Lq , L_s , L_q and W_s in two cases and comparing all these four characteristics in each case. If there is only one queue then the system considered as $M/M/S$ queuing system, in this case we will use multi server queuing model to find Lq , L_s , Wq and W_s .

If there are S queues in the system the queuing system considered as S isolated $M/M/1$ queuing systems, in this case we will use single server queuing model to find Lq , L_s , Wq and W_s in this case the customer's arrival rate become λ/S .

The following data is collected through questionnaires and survey for a supermarket which has 3 checkout counters. The data has been divided into 3 slots of morning, afternoon and evening.

Expected total cost for $M/M/1$ and $M/M/S$ model:

Service level is the function of two conflicting costs:

- i) Cost of offering the service to the customers
- ii) Cost of delay in offering service to the customers

Economic analysis of these costs helps the management to make a trade-off between the increased costs of providing better service and the decreased waiting time costs of customers derived from providing that

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Vol. 6, Issue 9, September 2017

service. Expected service cost $E(SC)=sC_s$ Where s is number of servers and C_s is service cost of each server

Expected waiting cost in the system $E(WC)=L_s C_w$

Where L_s =Expected number of customers in the system

And C_w =cost of waiting by the customer

Expected total cost in the case multi queue – multi server model (i.e. S individual $M/M/1$ models) $E(TC)=S(E(SC)+E(WC))=s(C_s + L_s C_w)$

Expected total cost in the case single queue – multi server model (i.e. $M/M/s$ model)

$E(TC)=E(SC)+ E(WC)= sC_s + L_s C_w$

IV. ANALYSIS OF DATA

Present scenario:

Here, the customers join the queue by themselves. The data were collected for $s=3$ checkout counters. So, there are 3 queues and the customers select their own queue. In this case, customers in any queue switch to shorter queue (jockey behavior of queue).

The following calculations are made using the performance measures of $M/M/1$ queuing model having 3 isolated checkout counters.

Time slot	Arrival rate λ	Service rate μ	L_s	L_q	W_s	W_q
10am to 2pm	13.33	20	2	1.33	9 min	6 min
2pm to 6pm	6.67	20	0.5	0.1667	4.5 min	1.5min
6pm to 10pm	16.67	20	5	4.1667	18 min	15min

To find the expected total cost assuming that service cost is Rs. 200 and waiting cost is Rs.250 in each case, $E(TC)=S(E(SC)+ E(WC))=s(C_s + L_s C_w) = 2475$

Proposed model:

There are 3 checkout counters but only one queue and there is a machine which gives indication to the next customer that which queue should be chosen. As soon as one counter gets vacant, it shows on the machine and the next customer joins that queue.

The calculations are made using the performance measures of $M/M/s$ queuing model having $s=3$ checkout counters.

Time slot	Arrival rate λ	Service rate μ	L_s	L_q	W_s	W_q
10am to 2pm	40	20	2.8889	0.8889	4.33min	1.33min
2pm to 6pm	20	20	1.0455	0.0455	3.14 min	0.14min
6pm to 10pm	50	20	6.0112	3.5112	7.21 min	4.21min

Expected total cost in the case single queue – multi server model (i.e. $M/M/s$ model)

$E(TC)=E(SC)+ E(WC)= sC_s + L_s C_w = 1578.8$

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Vol. 6, Issue 9, September 2017

V. CONCLUSION

In this paper, we conclude that single queue multi server is better in comparison to multi queue multi server. The waiting time of customers waiting in the queue is reduced almost 3 times to the previous one. We also proved that expected total cost is less in case of single queue multi server as compared to multi queue multi server model.

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